Wimborne Area Schools

# Framework for Progression in Mathematical Calculation

St John's First School, Merley First School, Pamphill First School, Wimborne First School, Witchampton First School, Allenbourn Middle School With thanks to: Henbury View First School and Sturminster Marshall First School

#### Framework for Progression in Mathematical Calculation

#### Introduction

This document and the progressions for developing effective methods of calculation have been compiled by a consortium of first and middle schools from the Wimborne area and adopted as the agreed route to efficient calculation.

This framework details the key written methods of mathematical calculation to be taught. Its purpose is to promote a consistent and progressive approach to the teaching of mathematical calculation skills, in line with the expectations of the 2014 Mathematics curriculum. Although the main focus of this policy is on the progression to pencil and paper procedures it is important to recognise that the ability to calculate mentally underpins all calculation. Written calculation methods are not a replacement for mental calculation but structures to enable more complex calculations to be carried out efficiently. In every written method there is an element of mental processing. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately. Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills, learned in working with a range of manipulative equipment, which provide the foundation for jottings and informal written methods of recording. These mental skills lead on to more formal written methods of calculation. Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When introducing a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept then, as competence increases, larger and more complex numbers can be tackled. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. The transition between years should not be hurried as not all children will be ready to move on to the next stage at the same time. Progression to the next stage should be made when mastery of the current stage is evident. Progression should be based on attainment across the mathematics curriculum and not just in the calculation processes. Judgements of mastery should be based on evidence gathered in routine classroom assessments and tasks of the pupils competence and accuracy in applying methods learned.

The long-term aim for our children is to have, and be able to select from, a variety of efficient (accurate, reliable and quick) methods of calculation that are appropriate to solve a range of calculation problems.

They should do this by always asking themselves:

- 'Can I do this in my head?'
- · 'Can I do this in my head using drawings or jottings?'
- · 'Do I need to use a written method?'

	Addition Mentally (Including Jottings)	Addition Written	
Year	represent and use number bonds within 20	• read, write and interpret mathematical statements involving addition (+) and	
1	<ul> <li>represent number pairs to = 10</li> <li>use number pairs to = 10 to work out number pairs to = 20</li> <li>represent doubles up to double 10</li> <li>represent number bonds to make 3, 4, 5, 6, 7, 8, 9 in all ways using addition</li> <li>use number bonds to make 3, 4, 5, 6, 7, 8, 9</li> </ul>	<ul> <li>equals (=) signs</li> <li>5 + 7 =</li> <li>solve missing number problems (using numbers up to 20)</li> </ul>	
	<ul> <li>add one-digit and two-digit numbers to 20, including zero</li> </ul>	7 = + 2	
	- using concrete objects (including exchanging Tens & Ones)		
	- using pictorial representations $(\bigcirc \circ ) + (\circ \circ) = 5$ $(\bigcirc \circ) $	Adding $6+33$ $1 + 4 + 5 + 7 + 8 + 9 + 10$ $3+45 - 7 + 8 + 9 + 10$ $3+45 - 7 + 8 + 9 + 10$	



	Division Mentally (Including Jottings)	Division Written
Ī	• calculate the answer to division problems (with the support of the teacher)	(mental with jottings only)
	(Using humbers up to 20)	
	- using pictorial representations	
	- using arrays	
	15 ÷ 3 = 5	

	Addition Mentally (Including Jottings)	Addition Written	
Year 2	<ul> <li>recall and use addition facts to 20 fluently, and derive and use related facts up to 100</li> </ul>	<ul> <li>add a two-digit number and ones (using numbers up to 100) (no bridging the tens, bridging the tens)</li> </ul>	
2	<ul> <li>recall number pairs to = 10</li> <li>recall number pairs to = 20</li> <li>derive number pairs to = 100 in multiples of 10</li> <li>derive number pairs to = 100 e.g. 43 + ? = 100</li> <li>recall doubles up to double 10</li> <li>recall number bonds to make 3, 4, 5, 6, 7, 8, 9 in all ways using addition</li> <li>derive additions of multiples of 10</li> <li>add a two-digit number and ones (using numbers up to 100)</li> <li>add a two-digit number and tens (using numbers up to 100)</li> <li>add two two-digit numbers (using numbers up to 100)</li> </ul>	$18+7=$ $18+7=$ $18 + 7 =$ $18$ $\frac{+7}{25}$ $18$ $19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $18$ $\frac{+7}{25}$ $18$ $19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $18$ $\frac{+7}{25}$ $18$ $19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $18$ $\frac{+7}{25}$ $18$ $19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $18$ $19 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $18$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 20 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 21 \ 22 \ 23 \ 24 \ 25$ $10 \ 21 \ 23 \ 24 \ 25$ $10 \ 21 \ 23 \ 24 \ 25$ $10 \ 21 \ 25 \ 25 \ 25 \ 25 \ 25 \ 25 \ 25$	
	add three one-digit numbers	$34 \pm 20 =$	
	The second condition of the second seco	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	- using pictorial representations $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	• add two two-digit numbers (using numbers up to 100) (no bridging the tens, bridging the tens) 34 + 23 = 57 44 44 54 57 34 44 54 57 NOTE: See text box above. + 23 + 29	

Year	Subtraction Mentally (Including Jottings)	Subtraction Written		
2	<ul> <li>recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100</li> </ul>	<ul> <li>subtract a two-digit number and ones (no bridging the tens, bridging the tens)</li> </ul>		
	<ul> <li>use number bonds to make 3, 4, 5, 6, 7, 8, 9 to work out subtractions i.e. use reversals.</li> <li>represent 5 + 3 = 8, therefore 8 - 3 = 5</li> <li>recall halves of even numbers up to 20</li> </ul>	15-7= $-1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -$		
	subtract a two-digit number and ones	15 Calculation to be set out in columns to support the 7 understanding of place value.		
	subtract a two-digit number and tens	However, this should be solved using a range of strategies,		
	subtract two two-digit numbers	e.g. Number line, Dienes, Numicon, diagrams and jottings.		
	subtract three one-digit numbers	Partitioning & exchanging should be included.		
	- using concrete objects (including exchanging Tens & Units)			
		• Subtract a two-digit number and tens		
		-10 -10 47 57 67		
	- using pictorial representations	67 - 20 NOTE: See text box above.		
		<ul> <li>subtract two two-digit numbers (no bridging the tens, bridging the tens)</li> <li>23 - 12 =</li> </ul>		
	- mentally (with jottings)			
	54 - 27 $54 - 20 = 34$			
	- 4 = 30	23 43		
	-3 = 27	- 12 - 27		

	Multiplication Mentally (Including Jottings)	Multiplication Written	
Year	• recall and use multiplication facts for the 2, 5 and 10 multiplication tables	write calculations using the multiplication (x) and equals (=) signs	
2	recognise odd and even numbers	4 x 5 = 20	
	<ul> <li>calculate mathematical statements for multiplication within the multiplication tables</li> </ul>	<ul> <li>calculate mathematical statements for multiplication within the multiplication tables</li> </ul>	
	- using materials	- using arrays	
	- using mental methods (with jottings)	$3 \times 5 = 15$ $4 \times 5 =$ $5 \times 5 \times$	
	5, 10, 15, 20, 25	5+5+5=20	
	Division Mentally (Including Jottings)	Division Written	
	<ul> <li>recall and use division facts for the 2, 5 and 10 multiplication tables</li> <li>use 2x table to divide by 2, counting in multiples or reversing the calculation</li> <li>use 10x table to divide by 10, counting in multiples or reversing the calculation</li> <li>use 5x table to divide by 5, counting in multiples or reversing the calculation</li> <li>calculate mathematical statements for division within the multiplication tables</li> <li>using materials</li> <li>using mental methods (with jottings)</li> </ul>	<ul> <li>write calculations using the division (÷) and equals (=) signs</li> <li>30 ÷ 2 = 15</li> <li>calculate mathematical statements for division within the multiplication tables (i.e. no remainders)</li> <li>using arrays</li> <li>15 ÷ 5 = 3</li> <li>- using repeated addition</li> </ul>	
	6 6	15÷5= 5 10 15	

	Addition Mentally (Including Jottings) Addition Written				
Year	add a three-digit number and ones (using numbers up to 1000)	• add numbers with up to three digits, using formal written methods of			
3	<ul> <li>add a three-digit number and tens (using numbers up to 1000)</li> </ul>	columnar addition			
	<ul> <li>add a three-digit number and hundreds (using numbers up to 1000)</li> </ul>	(No exchanging, exchanging ones to tens, exchanging tens to hundreds)			
	(no bridging the hundreds, bridging the hundreds)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			
	Subtraction Mentally (Including Jottings)     subtract a three-digit number and ones	Subtraction Written     subtract numbers with up to three digits, using formal written methods of			
	<ul> <li>subtract a three-digit number and tens</li> </ul>	columnar subtraction			
subtract a three-digit number and hundreds		(No exchanging, exchanging tens to ones, exchanging hundreds to tens)			
	(no bridging the hundreds, bridging the hundreds)	$\begin{array}{cccccccccccccccccccccccccccccccccccc$			

Multiplication Mentally (Including Jottings)		Multiplication Written		
recall and use multiplication facts for	r the 3, 4 and 8 multiplication tables	• write and calculate mathematical statements for multiplication using the		
<ul> <li>write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental strategies</li> </ul>		multiplication tables that they know, including for two-digit numbers times one-digit numbers progressing to formal written methods (No exchanging, exchanging ones to tens, exchanging tens to hundreds)		
$35 \times 4  70 \times 2 = 140  35 \times 4  20  20$	35 x 4 3 x 4 = 12 5 x 4 = 20 140	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
Division Mentally	(Including Jottings)	Division Written		
<ul> <li>recall and use division facts for the 3</li> </ul>	3. 4 and 8 multiplication tables	write and calculate mathematical statements for division using the		
		multiplication tables that they know, including for two-digit numbers divided by one-digit numbers (no remainders, remainders)		
27 ÷ 3 3, 6, 9, 12, 15, 18, 21,	24, 27 9 x 3 = 27			

	Addition Mentally (Including Jottings)	Addition Written	
Year 4	It would be helpful if children could begin to add 4 digit numbers mentally where appropriate, as Yr 3 add 3 digit and Yr 5 add 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit + 1s 4 digit + 10s 4 digit + 100s	<ul> <li>add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate</li> <li>(Exchanging ones to tens, exchanging tens to hundreds, exchanging hundreds to thousands)</li> <li> <sup>2</sup> 3 4 5         <sup>2</sup> 3 5 5         <sup>2</sup> 8 5 5         <sup>4</sup> 4 6         <sup>4</sup> 4 5 6         <sup>4</sup> 3 7 6 6         <sup>2</sup> 3 9 1         <sup>2</sup> 8 1 1         <sup>1</sup> 1</li></ul>	
	Subtraction Mentally (Including Jottings)	Subtraction Written	
	It would be helpful if children could begin to subtract 4 digit numbers mentally where appropriate, as Yr 3 subtract 3 digit and Yr 5 subtract 5 digit. (no bridging, bridging) (include jottings where necessary) 4 digit - 1s 4 digit - 10s 4 digit - 100s Finding the difference between 2 larger numbers close together can be taught by counting/jumping up from the lowest. 1000 – 998 = 2	<ul> <li>subtract numbers with up to 4 digits using the formal written methods of columnar and subtraction where appropriate</li> <li>(Exchanging tens to ones, exchanging hundreds to tens, exchanging thousands to hundreds)</li> <li> <sup>31</sup> <sup>21</sup> <sup>11</sup> <sup>1</sup> <sup>1</sup> <sup>1</sup> <sup>31</sup> <sup>21</sup> <sup>31</sup> <sup>3</sup></li></ul>	
	Multiplication Mentally (Including Jottings)	Multiplication Written	
	• recall multiplication facts for multiplication tables up to $12 \times 12$ (6,7,9,11,12) • use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers $35 \times 6$ $35 \times 6$ 30 $35 \times 6 = 18$ 30 30 $35 \times 6 = 30$ 210	• multiply two-digit and three-digit numbers by a one-digit number using formal written layout $ \begin{array}{r} 2  4 \\                                  $	

Division Mentally (Includeing Jottings)	Division Written
<ul> <li>recall division facts for multiplication tables up to 12 × 12</li> </ul>	write and calculate mathematical statements for division using the
use place value, known and derived facts to divide mentally	a 2 digit number. (no remainders, remainders)
240 ÷ 6 =	846 ÷ 6 =
$24 \div 6 = 4$ $240 \div 6 = 40$	- 600 100 x 6 Answer to each
divide by 1	$\begin{array}{cccc} - 60 & 10 \times 6 \\ - 60 & 10 \times 6 \end{array} \qquad \qquad$
multiply together three numbers	- 60 10 x 6 required. See Yr3.
	- 60 10 x 6
	- 6 1 x 6

	Addition Mentally (Including Jottings)	Addition Written
Year	<ul> <li>add numbers mentally with increasingly large numbers</li> </ul>	add whole numbers with more than 4 digits, including using efficient
5	10573 + 3200 = 13773 • add numbers to 2 decimal places 9.58 + 3.08 = 12.66	written methods (columnar addition) 44388 $+ 5896$ $-50284$ $1111$ • add numbers to 3 decimal places 32.148 $+ 9.738$ $-41.886$ $1 1$
	Subtraction Mentally (Including Jottings)	Subtraction Written
	<ul> <li>subtract numbers mentally with increasingly large numbers</li> </ul>	<ul> <li>subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction)</li> </ul>
	64501 – 4300 = 60201	
	<ul> <li>subtract numbers to 2 decimal place</li> <li>7.47 - 3.15 = 4.32</li> </ul>	$ \begin{array}{c} 5 & 13 & 1 \\ 6 & 4 & 6 & 7 \\ - & \underline{2 & 6 & 8 & 4} \\ 3 & 7 & 8 & 3 \end{array} \\ \hline & \text{subtract numbers to 3 decimal places} \\ & \$ & 148 \\ - & \underline{9.738} \\ & 22.410 \end{array} $

	Multiplication Mentally (Including Jottings)		Multiplication Written
	<ul> <li>multiply and divide numbers mentally drawing upon known facts         45 x 6 =             45 x 2 = 90, 90 x 3 (9x3x10)             = 270             38 x 15 =             38 x 10 = 380 + 190 (1/2 of 380)             = 570             solve problems involving multiplication where larger numbers are used by             decomposing them into their factors             350 x 25 = 7 x 5 x 10 x 5 x 5      </li> <li>multiply whole numbers and those involving decimals by 10, 100 and 1000             234.7 x 10 becomes             2347.0             9067.43 x 100 becomes             906743.0      </li> </ul>		<ul> <li>multiply numbers up to 4 digits by a one-digit number using an efficient written method <ul> <li>4346</li> <li><u>x 8</u></li> <li><u>34768</u></li> <li><u>234</u></li> </ul> </li> <li>multiply numbers up to 4 digits by a two-digit number using long multiplication 4115 x 25 <ul> <li>4115</li> <li><u>x 25</u></li> <li><u>1</u></li> <li><u>82300</u> (x10, x2)</li> <li><u>20575</u></li> <li><u>102875</u></li> <li>recognise and use square numbers and cube numbers, and the notation for squared () and cubed ()</li> <li>2<sup>2<sup>2</sup> = 2 x 2 = 4</sup></li> <li>3<sup>3</sup> = 3 x 3 x 3 = 27</li> </ul> </li> </ul>
	Division Ment	ally (Including Jottings)	Division Written
<ul> <li>divide numbers mentally drawing upon known facts</li> <li>multiply and divide whole numbers involving decimals by 10, 100 and 1000 234.7 ÷ 10 becomes 23.47</li> <li>When dividing by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the right for each multiple of 10</li> <li>becomes 90.6743</li> </ul>		y upon known facts ers involving decimals by 10, 100 and 1000 When dividing by 10 and multiples of 10, the decimal point remains fixed and the number moves 1 place to the right for each multiple of 10	• divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context $72 \div 5$ $5 \frac{14 \text{ r } 2}{5 2 2}$ $72$ $1368 \div 9$ $9 \frac{152}{5 4 1}$ $1368$

	Addition Mentally (Including Jottings)		Addition Written
Year 6	•	add numbers mentally with increasingly large numbers	<ul> <li>add whole numbers with more than 4 digits, including using efficient written methods (columnar addition) (See Year 5 for examples)</li> </ul>
		Subtraction Mentally (Including Jottings)	Subtraction Written
subtract numbers mentally with increasingly large numbers     (5		subtract numbers mentally with increasingly large numbers	<ul> <li>subtract whole numbers with more than 4 digits, including using efficient written methods (columnar subtraction)</li> <li>(See Year 5 for examples)</li> </ul>
	Multiplication Mentally (Including Jottings)		Multiplication Written
	•	perform mental calculations, including with mixed operations and large numbers	<ul> <li>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication</li> </ul>

Division Mentally (Including Jottings)	Division Written     divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	
<ul> <li>perform mental calculations, including with mixed operations and large numbers</li> </ul>		
	2666 ÷ 42 Romaindar 62 r20	
	42)2666	10 x 42= 420
	252	$2 \times 42 = 84$
	146	4 x42 = 168
	126	5 x 42 = 210
	20	6 x 42 = 252
	Fraction 63 20/42	
	42)2666 <u>252</u> 146	Use doubling/halving to find other multiples.
	<u>126</u> 20	See Appendix 1 for Bar and Double Number Line representations
	Decimal <u>63.47</u> 42)266 <i>00</i>	
	- <u>252</u>	
	-126	
	200	
	- <u>168</u>	
	320	
	- <u>294</u>	
	26	

#### Tools to aid multiplication in division calculations





#### **Double Number Line Method**



This method of multiplication uses know or easily computable facts, by doubling and halving or addition, to construct the data needed to carry out the calculation. There is no need to calculate all values from 1 times to 10 times, only those needed.